Verifying the Mathematical Library of a UAV Autopilot with Frama-C

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Context

Formal methods

- Verification techniques based on mathematical models
- Recommended in avionics with DO-178C and DO-333 standards
- Example: abstract interpretation, deductive methods, model-checking

The goals of my PhD

- Define verification processes that use formal methods,
- Apply these methods to a drone autopilot: Paparazzi.

First results

Analysis of a mathematical library of Paparazzi:

- Using Frama-C,
- Checking for the absence of runtime errors,
- Verification of some functional properties,
- Without modifying the code.

Frama-C

Frama-C

Frama-C is a C code analysis tool

- Developed by CEA and INRIA,
- Modular, which supports different analysis methods
  ex: static analysis with EVA or dynamic analysis with E-ACSL.

Verification process of a C program using Frama-C:

1. Code specification with **ACSL** (*ANSI C Specification Language*),
2. Generation of the abstract syntax tree of the analyzed code,
3. Analysis of the tree by the plugins
   $\Longrightarrow$ Verify if the specification is respected.

Note: the tree analysis can be performed by several plugins.

Some Frama-C plugins

RTE (*RunTime Errors*):

- Adds assertions in the code,
- Allows to verify runtime errors
  ex: division by 0, overflows ...

WP (*Weakest Precondition*)

- Uses a calculation of weakest preconditions,
- Interfaced with Why3 to verify goals with automatic provers (Alt-Ergo, Z3, CVC4).

EVA (*Evolved Value Analysis*)

- Based on static analysis by abstract interpretation methods,
- Computes domains of values for each variable in the program.

Paparazzi

Presentation

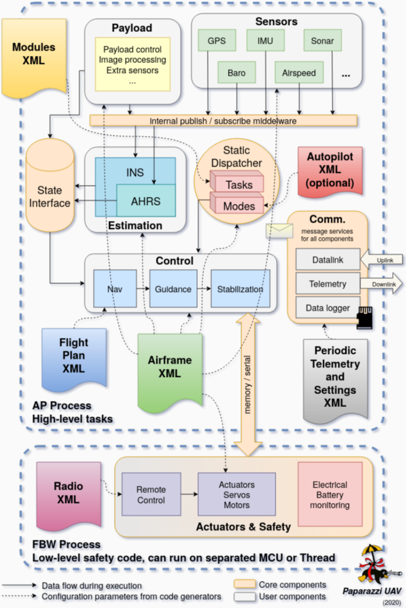
Paparazzi is an autopilot for micro-drones

- Developed at ENAC since 2003,
- Open-Source under GPL license.

Complete drone control system:

- Offers the control software part,
- Also offers some designs of hardware components,
- Supports for ground and aerial vehicles,
- Supports for simultaneous control of several drones.

Flight system architecture



Library studied

`pprz_algebra` : mathematical algebra library coded in C ($\sim$ 4 000 loc)

The library contains:

- The definition of a representation of `vectors`,
- Different representations of vector rotations,
  rotation matrices, Euler angles, quaternions
- Elementary operations,
  ex: addition of vectors, computation of the rotation of a vector, normalization of a quaternion . . .
- Conversion functions between these different representations.

Note: Each representation/function has a fixed point (`int`) and floating-point version (for `float` and `double`).

Absence of runtime errors

Absence of runtime errors

There are different types of runtime errors:

- Dereferencing an invalid pointer,
- Division by 0,
- Overflows,
- Non finite float value,
- ...

Goals: To determine the minimum contracts for the functions of the library in order to guarantee the absence of runtime errors.

Process :

- Analyze the code with Frama-C using RTE and WP plugins.
- Deduce the missing information in the contract.

Analysis with Frama-C and the RTE plugin

Analysis of the instruction:

```
c->x = a->x * b->x;
```

Frama-C finds 2 potential errors!

- Pointers might not be valid.

```
/*@ assert rte: mem_access: \valid(&c->x); */
/*@ assert rte: mem_access: \valid_read(&a->x); */
/*@ assert rte: mem_access: \valid_read(&b->x); */
```

$\Longrightarrow$ Require the validity of pointers as a precondition.

- The values are not bounded.

```
/*@ assert rte: signed_overflow: -2147483648 ≤ a->x * b->x; */
/*@ assert rte: signed_overflow: a->x * b->x ≤ 2147483647; */
```

$\Longrightarrow$ Determine bounds which guarantee the absence of overflows.

Interval Computation

Computation of the range of possible values for `a->x` and `b->x` to ensure the absence of RTE in the instruction:

```
c->x = a->x * b->x;
```

The intervals must be equal.

We try to determine the maximum bound M such that:

If `a->x`$\in [-M; M]$

`b->x`$\in [-M; M]$

Then, `c->x` $\in$ $[\texttt{INT\_MIN}; \texttt{INT\_MAX}] \subseteq [-\texttt{INT\_MAX}; \texttt{INT\_MAX}]$

We can easily deduce that $M := \sqrt{\texttt{INT\_MAX}}$

Example of final contract for the function `int32_quat_comp`

```c
#define SQRT_INT_MAX4 23170 // 23170 = SQRT(INT_MAX/4)

/*@
  requires \valid(a2c);
  requires \valid_read(a2b);
  requires \valid_read(b2c);
  requires bound_Int32Quat(a2b, SQRT_INT_MAX4);
  requires bound_Int32Quat(b2c, SQRT_INT_MAX4);
  requires \separated(a2c, a2b) && \separated(a2c, b2c);
  assigns *a2c;
*/
void int32_quat_comp(struct Int32Quat *a2c,
                     struct Int32Quat *a2b,
                     struct Int32Quat *b2c)
```

Notes

EVA and WP had to be associated to verify the absence of RTE.

- WP is overloaded when accessing values by reference,
- EVA cannot verify loop variants and invariants.

$\Longrightarrow$ The same problem has been raised in the thesis of V. Todorov.

The `real` **arithmetic model** (real in the mathematical sense) has been used to verify floating-point version of the functions.

The `real` model guarantees :

- The absence of division by 0,
- The lack of dereference of invalid pointers.

But the absence of overflows is not verified.

Functional verification

Functional verification

Functional verification

Offer guarantees on the behavior or the result of a function.

Example: *Functional properties for square root function*

```
/*@
  requires x >= 0;
  ensures \result >= 0;
  ensures \result * \result == \old(x);
  assigns \nothing;
*/
float sqrt(float x);
```

Note : Verifying these properties is only possible with the *real* model.

The functional verification was only done for some floating-point functions.

Specifying the functional properties of the library

Functional properties must be expressed in the ACSL logic.

First, it is necessary to define:

- **Types**,
  ex : `RealVect3`, `RealRMat`, `RealQuat`.
- **Elementary functions**,
  ex: addition of vectors, rotation of a vector...
- **Conversion functions** between certain representations,
  ex: Converting a quaternion to a rotation matrix.
- **Lemmas**.
  ex: The conversion function produces the same rotation, ...

Finally, the functional properties are expressed in the form of predicates:

- M is a rotation matrix: $M.M^t = I$
- ...

Example

Spécification of the function `float_rmat_of_quat`.

```
/*@
    requires \valid(rm);
    requires \valid_read(q) && finite_FloatQuat(q);
    requires unitary_quaternion(q);
    requires \separated(rm, q);
    ensures rotation_matrix(l_RMat_of_FloatRMat(rm));
    ensures special_orthogonal(l_RMat_of_FloatRMat(rm));
    ensures l_RMat_of_FloatRMat(rm) ==  l_RMat_of_FloatQuat(q);
    assigns *rm;
*/
void float_rmat_of_quat(struct FloatRMat *rm,
                        struct FloatQuat *q)
```

Computation errors in the function `float_rmat_of_quat`.

Problem :

WP and EVA could not verify some properties.

After analyzing the code:

- The defined functions and the C code do not always produce the same result, despite using the *real* model.
- The C code uses a constant `M_SQRT2` to represent $\sqrt{2}$,
- When the calculations are unfolded, we always get:

$$\texttt{M\_SQRT2} * \texttt{M\_SQRT2} \neq 2$$

$\Longrightarrow$ **The code therefore always produces calculation errors.**

Proposed code modification

Original code:

```
#define M_SQRT2        1.41421356237309504880
const float _a = M_SQRT2 * q->qi, _b = M_SQRT2 * q->qx;
(...)
const float a2_1 = _a * _a - 1;
(...)
RMAT_ELMT(*rm, 0, 0) = a2_1 + _b * _b;
(...)
```

Modified code:

```
const float _a  = q->qi , _b  = q->qx ;
const float _2a = 2 * _a, _2b = 2 * _b;
(...)
const float a2_1 = _2a * _a - 1;
(...)
RMAT_ELMT(*rm, 0, 0) = a2_1 + _2b * _b;
(...)
```

Summary of the functional verification

Functional verification offers guarantees on the behavior of a function.

The code modification proposed for the function `rmat_of_quat` :

- Eases contract verification.
- Reduces computation errors at runtime,
- Does not modify the number of operations,

Using the `real` model:

- Used to verify that the code is correct in a mathematical sense,
- Offers no functional guarantee during execution.

Conclusion

Conclusion

Summary :

- Verification of the absence of runtime errors in the library,
- Verification of functional properties on some floating-point functions.

$\Longrightarrow$ Approximately 3,500 lines of annotation.

`gitlab.isae-supaero.fr/b.pollien/paparazzi-frama-c`

Perspectives:

- Verification of calls to library functions,
- Verifying the floating-point library without the `real` model,
- Verifying the Paparazzi flight plan generator.

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Thank you for your attention

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