Formal Verification of an UAV autopilot

Static analysis and Verified Code Generation

Baptiste Pollien[1]

Thesis supervised by
Christophe Garion[1], Gautier Hattenberger[2], Pierre Roux[3] and Xavier Thirioux[1]

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[1]ISAE-SUPAERO - [2]ENAC - [3]ONERA, Toulouse, France

Critical systems

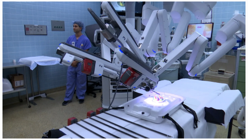
Systems which must be highly reliable and where any bugs can be costly or life-endangering.

They can be found in several domains:

Space

Avionics

Automotive



Medical

Nuclear

Autonomous Drone

Introduction
Development of a system: 3 main steps

Specification

Definition of:

- functional needs,
- guarantees required (safety and security),
- constraints (time, consumption...).

Implementation

Refinement of the specification into a system.

Verification

Ensure that the system matches its specification.

Crucial step, especially for **critical systems**.

Traditional verification and validation techniques:

- Code review,
- Tests.

"Program testing can be a very effective way to show the presence of bugs, but it is hopelessly inadequate for showing their absence.", Edsger Dijkstra.

How to be more confident in the absence of errors?

$\Longrightarrow$ A solution is to use Formal Methods.

Formal methods

- Verification techniques and tools based on mathematical models and proofs,
- Offer stronger guarantees than test.
- Examples: abstract interpretation, deductive methods, model-checking.

Industrial use:

- Used in several domains: aerospace, automotive, medical, cybersecurity, etc.
- Recommended in avionics with DO-178C and DO-333 standards.

Limitations

- Verification tools not always scalable on large projects,
- Applied by engineers not trained in formal methods.

Goals of this thesis:

- Review verification processes using formal tools,
- Apply them on critical components,
- Ensure that these processes can be used on existing projects.

Thesis realised in the context of the **Concorde Project**.

Concorde Project

- Research project supported by Defense Innovation Agency (AID).

Goals: Propose methods for the analysis and design towards the certification of future drones systems and their operations.

$\Longrightarrow$ Apply the verification processes on **critical components** of a **drone autopilot**.

Case study: Paparazzi UAV autopilot, developed at ENAC.

Paparazzi is a good candidate for testing if formal methods are usable/efficient as

- The autopilot has been developed:
  - without verification purpose,
  - by good programmers,
  - using classic C idioms in the code (pointers, etc).
- The code base is sizable ($\sim$ 350,000 lines of code).

This thesis focuses on 2 critical components:

- A mathematical library used by the control system.
  $\Longrightarrow$ Verified using static code analysis.
- A flight plan generator producing embedded C code.
  $\Longrightarrow$ Verified using code generation verification techniques.

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Paparazzi is an autopilot for micro-drones

- Developed at ENAC since 2003,
- Open-Source under GPL license.

Complete UAV control system:

- Control embedded software part,
- Design of some hardware components,
- Support for ground and aerial vehicles,
- Support for simultaneous control of several drones.

Paparazzi GCS connected to 3 drones [1]

[1] Pascal Brisset and Gautier Hattenberger. "Multi-UAV control with the Paparazzi system". In: *HUMOUS 2008*. Brest, France

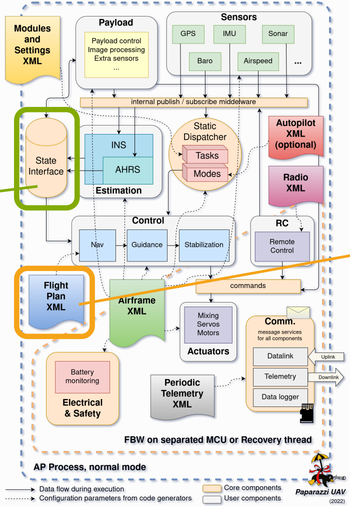
Paparazzi

Flight system architecture

State Interface

Black board interface:

- Collects data from sensors.
- Converts automatically the data between different representations, provided by a mathematical library.



Flight Plan

- Defines the behaviour of the drone once launched.
- Flight Plan Generator that converts XML flight plans into embedded C code.

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Mathematical Library

Library studied

`pprz_algebra` : mathematical algebra library coded in C ($\sim$ 3 200 lines of code)

Library used for UAV state representations, in particular **attitude and speed representations**.

The library contains:

- The definition of a representation of `vectors`,
- Different representations of vector rotations,
  rotation matrices, Euler angles, quaternions.
- Elementary operations,
  ex: addition of vectors, computation of the rotation of a vector, normalisation of a quaternion, etc
- Conversion functions between these different representations.

Note: Each representation/function has a fixed point (`int`) and floating-point versions (for `float` and `double`).

Data produced is used by the navigation system.

$\Longrightarrow$ **Any bug can lead to the crash of the program or produce invalid data.**

Existing C verification tools:

- **CBMC**, a model checker for C programs.
- **VST**, a set of tools and methods for the formal verification of C software.
- **Frama-C**, a workbench implementing several verification methods for C code.
- ...

Our objective: Ensure the correctness of the library using Frama-C, **without modifying the code**.

Frama-C

Frama-C

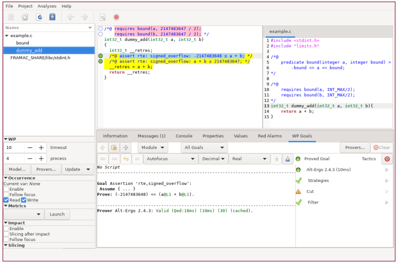
Frama-C is a C code analysis tool

- Mainly developed by CEA,
- Modular, which supports different analysis methods
  ex: static analysis with EVA or dynamic analysis with E-ACSL.

Verification process of a C program using Frama-C:

1. Code specification with **ACSL** (*ANSI C Specification Language*),
2. Generation of the abstract syntax tree of the analysed code,
3. Analysis of the tree by the plugins
   $\Longrightarrow$ Verify whether the specification is respected.

Note: the tree analysis can be performed by several plugins.



Frama-C GUI

Some Frama-C plugins

RTE (*RunTime Errors*):

- Adds assertions in the code,
- Allows to verify the absence of runtime errors
  ex: division by 0, overflows ...

WP (*Weakest Precondition*)

- Implements weakest precondition calculus,
- Interfaced with Why3 to verify goals with automatic provers (Alt-Ergo, Z3, CVC4).

EVA (*Evolved Value Analysis*)

- Based on static analysis by abstract interpretation methods,
- Computes domains of values for each variable in the program.

Absence of runtime errors

Absence of runtime errors

There are different types of runtime errors in C:

- Dereferencing an invalid pointer,
- Division by 0,
- Overflows,
- Non finite float value,
- ...

Goal: Determine the "minimal" contracts for the functions of the library in order to guarantee the absence of runtime errors.

Process :

- Analyse the code with Frama-C using RTE and WP plugins.
- Deduce the missing information in contracts.

Analysis with Frama-C and the RTE plugin

Analysis of the instruction:

```
c->x = a->x * b->x;
```

Frama-C finds 2 potential errors!

- Pointers might not be valid.
  - `/*@ assert rte: mem_access: \valid(&c->x); */`
  - `/*@ assert rte: mem_access: \valid_read(&a->x); */`
  - `/*@ assert rte: mem_access: \valid_read(&b->x); */`

  $\Longrightarrow$ Require the validity of pointers as a precondition.

- The values are not bounded.
  - `/*@ assert rte: signed_overflow: -2147483648 ≤ a->x * b->x; */`
  - `/*@ assert rte: signed_overflow: a->x * b->x ≤ 2147483647; */`

  $\Longrightarrow$ Determine bounds which guarantee the absence of overflows.

Example of final contract
The function int32_quat_comp

```c
#define SQRT_INT_MAX4 23170 // 23170 = SQRT(INT_MAX/4)

/*@
  requires \valid(a2c);
  requires \valid_read(a2b);
  requires \valid_read(b2c);
  requires \separated(a2c, a2b) && \separated(a2c, b2c);
  requires bound_Int32Quat(a2b, SQRT_INT_MAX4);
  requires bound_Int32Quat(b2c, SQRT_INT_MAX4);
  assigns *a2c;
*/
void int32_quat_comp(struct Int32Quat *a2c,
                     struct Int32Quat *a2b,
                     struct Int32Quat *b2c)
```

Notes

EVA and WP had to be associated to verify the absence of RTE.

- WP is overloaded when accessing values by reference,
- EVA cannot verify loop variants and invariants.

$\Longrightarrow$ The same problem has been raised in the thesis of V. Todorov[2].

The `real` arithmetic model (real in the mathematical sense) has been used to verify floating-point version of the functions.

The `real` model guarantees :

- The absence of division by 0,
- The lack of dereference of invalid pointers.

But the absence of overflows and rounding errors are not verified.

[2] Vassil Todorov. "Automotive embedded software design using formal methods". PhD Thesis. Université Paris-Saclay, Dec. 2020

Functional verification

Functional verification

Offer guarantees on the behavior or the result of a function.

Example: *Functional properties for square root function*

```
/*@  requires x >= 0;
     ensures \result >= 0;
     ensures \result * \result == \old(x);
     assigns \nothing;
*/
float sqrt(float x);
```

Using the `real` model:

- Offers no functional guarantee during execution.
- Used to verify that the code is correct in a mathematical sense.

How to specify the functional properties?

Functional properties must be expressed in ACSL logic.

First, it is necessary to define:

- **Types**,
  ex : `RealVect3, RealRMat, RealQuat`.
- **Elementary functions**,
  ex: addition of vectors, rotation of a vector...
- **Conversion functions** between representations,
  ex: Definition of the function $rmat\_of\_quat : \mathbb{H} \to M_{3,3}(\mathbb{R})$,

```
/*@
    logic RealRMat l_RMat_of_FloatQuat(struct FloatQuat *q) =
    [...]
*/
```

- **Lemmas...**

Specifying the functional properties of the library

Lemmas: Verify that the mathematical definitions are correct.
Ex: The conversion function produces the same rotation,

- Mathematically,

$$\forall q \in \mathbb{H}, \forall v \in \mathbb{R}^3,\ q(0, v)q^* = (0, \texttt{rmat\_of\_quat}(q).v)$$

Finally, the functional properties are expressed in the form of predicates:

- M is a rotation matrix: $M.M^t = I \wedge det\ M = 1$
- ...

Example: Specification of the function `float_rmat_of_quat`.

```
/*@
    requires ...
    ensures rotation_matrix(l_RMat_of_FloatRMat(rm));
    ensures l_RMat_of_FloatRMat(rm) ==  l_RMat_of_FloatQuat(q);
*/
void float_rmat_of_quat(struct FloatRMat *rm, struct FloatQuat *q)
```

Functional properties specified and verified in some `float` function contracts.

- Contracts and lemmas mainly verified **automatically** with solvers.
- Some lemmas had to be proven **manually** with Coq ($\sim$9% of the lemmas).

$\Longrightarrow$ Approximately 2,600 lines of ACSL annotations and 200 lines of Coq for 3,200 lines of code.

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Flight Plan Generator

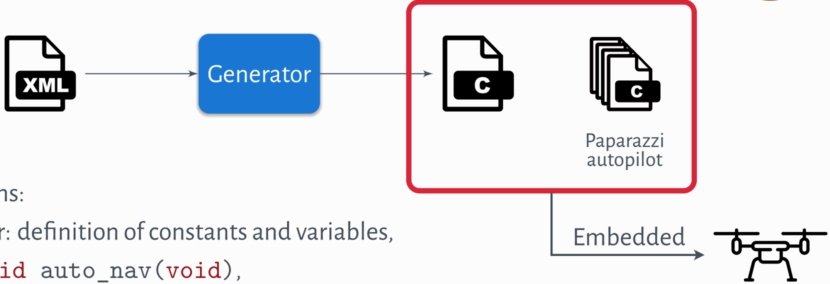
Flight Plan

The flight plan (FP)

- describes how the drone might behave when launched,
- is defined in a XML configuration file.

Example:

1. Wait until the GPS connection is set,
2. Take off,
3. Do a circle around a specific GPS position.
4. If battery is less than 20%: Go *home* and *land*.



The **generated C file** contains:

- The Flight Plan Header: definition of constants and variables,
- The main function: `void auto_nav(void)`,

$\Longrightarrow$ Compiled with the autopilot and embedded on the drone.

Function `auto_nav`:

- Called at 20 Hz,
- Sets navigation parameters for actuators.

Problems:

- The behaviour of flight plans is not formally defined.
- Does the `auto_nav` function always terminate?
- Generator is a complex software that generates embedded code.

$\Longrightarrow$ **Certified Compilation problem**

Solutions to similar problems

- CompCert: C compiler proved in Coq.
- Vélus: Lustre compiler proved in Coq.
- ...

Our objective: Develop a new verified flight plan generator in Coq.

Coq

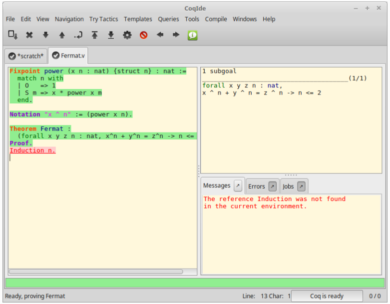
Coq

Coq is a proof assistant

- Development supported by Inria,
- Based on the Gallina language.

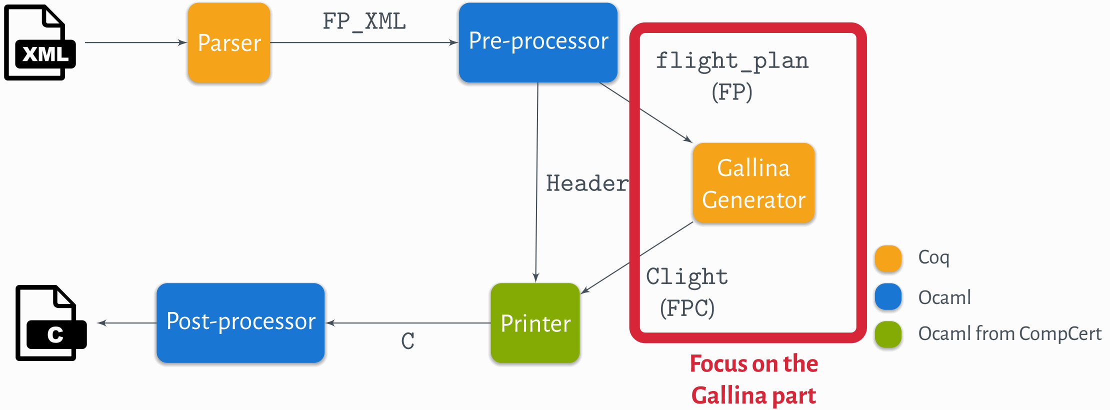
Software for writing and verifying formal proofs

- Proofs of mathematical theorems,
- Proofs of properties on programs.
  $\Longrightarrow$ Coq code can be extracted into OCaml code with guarantees.



CoqIDE, a vintage GUI for Coq

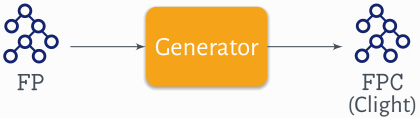
The New Verified Flight Plan Generator (VFPG)



Pre-processing: Manages included files, converts block names into indices...

Post-processing: Produces a compilable C code.

Overview of the semantics preservation proof



$$
\begin{tikzcd}
e_{fp} \arrow[rrr, squiggly, no head] \arrow[dd] & & & e_c \arrow[dd] \\
& o_{fp} \arrow[r, squiggly, no head] & o_c & \\
e'_{fp} \arrow[rrr, squiggly, no head] & & & e'_c
\end{tikzcd}
$$

FP semantics FPC semantics

Flight Plan Language

Flight Plan Structure in Gallina


```
Record flight_plan := {
    blocks: list fp_block
    excpts: list fp_exception;
    fb_deroutes: list fp_fb_deroute; (* New feature *)
}

Record fp_block := {
    id: block_id;
    excpts: list fp_exception;
    stages: list fp_stage;
}.

Inductive fp_stage :=
  | WHILE (cond: c_cond) (body: list fp_stage)
  | SET (var: var_name) (value: c_value)
  | CALL (fun: c_code)
  | DEROUTE (idb: block_id)
  | RETURN (reset: bool)
  | NAV (nav_mode: fp_nav_mode) (init: bool).

Record fp_exception := {
    cond: c_cond;
    id: block_id;
    exec: option c_code;
}.

Record fp_fb_deroute := {
    from: block_id;
    to: block_id;
    only_when: option c_cond;
}.
```

Example: Potential Execution of a Flight Plan

Flight Plan:

```
{| excpts : [ ],
   fb_deroutes : [ ],
   blocks : [
    {| id: 0, excpts : [ ],
         stages : [
             CALL "InitSensors()";
             WHILE "!GPSFixValid()" [ ];
             SET "home" "GPSPosHere()"]
    |};
    {| id: 1, excpts : [ ],
         stages : [
             NAV (TakeOff params) true;
             DEROUTE 10]
    |};
    ...  {| id: 10, ... |} ...
    ]
|}
```

Results of `auto_nav`:

Call	Current Block	Code Executed
1	0	`InitSensors()` `!GPSFixValid()` $\Uparrow$ `true`
2	0	`!GPSFixValid()` $\Uparrow$ `true`
3	0	`!GPSFixValid()` $\Uparrow$ `true`
⋮	⋮	⋮
9	0	`!GPSFixValid()` $\Uparrow$ `false` `home = GPSPosHere()`
10	1	`StartMotors()`
11	1	`TakeOffDone()` $\Uparrow$ `false`
12	1	`TakeOffDone()` $\Uparrow$ `false`
⋮	⋮	⋮
20	1	`TakeOffDone()` $\Uparrow$ `true` `Deroute` $\rightarrow 10$
21	10	...
⋮	⋮	⋮

Generator

Generator Function

```
Definition generate_flight_plan: flight_plan → res_generator
```

Inputs:

- Flight plan to convert.

Outputs:

- ```
  Variant res_generator :=
      | CODE (res: Clight.program * list err_msg)
      | ERROR (errs: list err_msg).
  ```
- Warnings and errors currently produced during the generation.
  - detect when there is a possible deroute that is forbidden,
  - detect when the flight plan has an incorrect size.
  ```

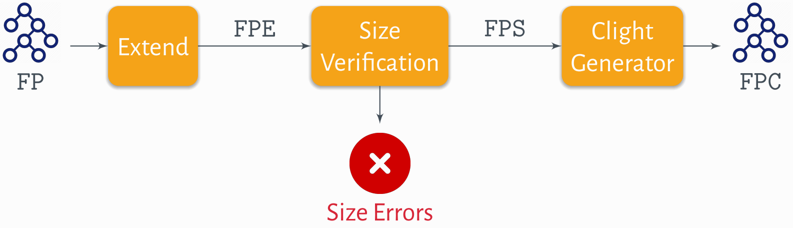
Example of generated C Code

Example of a flight plan:

```
{|  excpts: [],
    fb_deroutes: [],
    blocks: [
    {| id: 0,
        excpts: [],
        stages: [
            CALL "func1()";
            CALL "func2()"
        ]
    |}
    ]
|}
```

C code generated:

```
static inline void auto_nav(void) {
    switch (get_nav_block()) {
        case 0: // Block 0
            switch (get_nav_stage()) {
                case 0: // Stage 0
                    func1();
                case 1: // Stage 1
                    func2();
                default:
                case 3: // Default Stage
                    NextBlock();
                    break;
            }
            break;
        case 1: // Default Block
            GEN_DEFAULT_C_CODE()
    }
}
```



Extend Flight Plan:

- Index stages,
- Split NAV into NAV_INIT and NAV,
- Flatten stages contained in a WHILE stage.

Size verification:

- Check block indexing,
- Check that numbers of blocks and stages are less than 256,
- Check that block_id fields are 8 bits values.

Verification of the Generator

Generic Big Step Semantics for Flight Plans

Definition (fp_semantics)

A generic definition for the flight plan semantics.

```
Record fp_semantics: Type := FP_Semantics_gen {
    (** Environment for the semantics *)
    env: Type;
    (**  Properties stating if an env is an initial environment *)
    initial_env: env → Prop;
    (** Properties stating the execution of the auto_nav function *)
    step: env → env → Prop;
  }.
```

Instantiation of the semantics:

- FP semantics: semantics_fp,
- FPC semantics: semantics_fpc,
- FPE semantics: semantics_fpe,
- FPS semantics: semantics_fps.

Definition of the FP Semantics
A denotational semantics

Environment: `Definition fp_env := (fp_state * fp_trace).`

- `fp_state` the memory storing the execution **state** of the flight plan,
- `fp_trace`: the memory that can be modified by flight plan **external functions**.

Initial environment property noted $initial\_env\ e$

Step property noted $e \xhookrightarrow{FP} e'$.

- Defined as a function for early validation purposes: $e \xhookrightarrow{FP} e' := step\ e = e'$
- Interpretation of arbitrary C code.
  - **Hypothesis:** Arbitrary C code **terminates** and **does not modify** the FP state.
  - `Parameter eval: fp_env → cond → (bool * fp_env).`

- Inference rules for the `WHILE` stage.

$$\frac{e.\texttt{stages} = \texttt{WHILE}\,(cond, body) :: s \qquad \texttt{eval}\; e\; cond = (\texttt{true}, e') \qquad e'\{\texttt{stages} := body \mathbin{++} e.\texttt{stages}\} = e''}{e \overset{FP}{\hookrightarrow} e''}$$

$$\frac{e.\texttt{stages} = \texttt{WHILE}\,(cond, body) :: s \qquad \texttt{eval}\; e\; cond = (\texttt{false}, e') \qquad e'\{\texttt{stages} := s\} \overset{FP}{\hookrightarrow} e''}{e \overset{FP}{\hookrightarrow} e''}$$

- Inference rules for the `NAV` stage.

$$\frac{e.\texttt{stages} = \texttt{NAV}\,(mode, \texttt{true}) :: s \qquad e(\!|init\_nav\_code\; mode|\!) = e' \qquad e'\{\texttt{stages} := \texttt{NAV}\,(mode, \texttt{false}) :: s\} = e''}{e \overset{FP}{\hookrightarrow} e''}$$

$$\frac{e.\texttt{stages} = \texttt{NAV}\,(mode, \texttt{false}) :: s \qquad e \underset{nav}{\overset{FP}{\hookrightarrow}} e'}{e \overset{FP}{\hookrightarrow} e'}$$

fp_simulation describes if FP2 can simulate every behaviour of FP1.

```
Record fp_simulation (FP1 FP2: fp_semantics)
                     (match_envs: env FP1 → env FP2 → Prop): Prop := {
  match_initial_envs:
      ∀ (e1: FP1.env), initial_env e1 →
          ∃ (e2: FP2.env), initial_env e2 ∧ match_envs e1 e2;
  match_step:
      ∀ (e1 e1': FP1.env), step e1 e1' →
      ∀ (e2: FP2.env), match_envs e1 e2 →
          ∃ (e2': FP2.env), step e2 e2' ∧ match_envs e1' e2';
}.
```

Definition of a bisimulation relation between 2 semantics.

```
Inductive bisimulation (FP1 FP2: fp_semantics) : Prop :=
  Bisimulation (match_envs: env FP1 → env FP2 → Prop)
               (forward_simulation: fp_simulation FP1 FP2 match_envs).
               (backward_simulation: fp_simulation FP2 FP1 match_envs).
```

Correctness of the generator

Theorem (bisim_fp_fpc)

$$\forall\ \mathtt{fp}\ \mathtt{prog}\ \mathtt{warnings},\\ \mathtt{generator}\ \mathtt{fp} = \mathtt{CODE}\ (\mathtt{prog},\ \mathtt{warnings})\\ \rightarrow \mathtt{bisimulation}\ (\mathtt{semantics\_fp}\ \mathtt{fp})\ (\mathtt{semantics\_fpc}\ \mathtt{prog}).$$

This theorem states that the generator preserves the semantics.

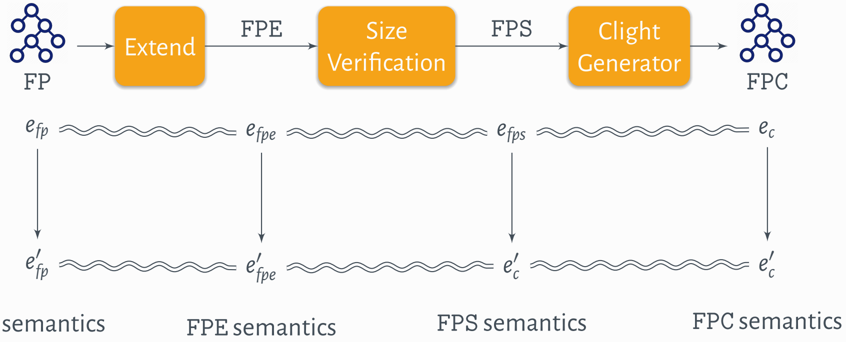
Forward simulation

FP behaviour is simulated by the Clight code.

Backward simulation

Every possible execution of the Clight code is described by the FP semantics.

Verification of the correctness theorem




```
Lemma compose_bisimulations:
    ∀ FP1 FP2 FP3, bisimulation FP1 FP2
                   → bisimulation FP2 FP3
                   → bisimulation FP1 FP3.
```

Interpretation of the **arbitrary C code**.

Hypothesis: Arbitrary C code **terminates** and **does not modify** the FP state.

New axioms to extend the **operational** Clight semantics.

$\Longrightarrow$ These axioms convert arbitrary C code into traces.

Note

These axioms can be **improved by modifying the generation** of the arbitrary C code.

Classic logic axioms from Coq standard library:

excluded middle, proof irrelevance and functional extensionality.

Constrained by the previous generator:

Input language, C code generated...

Split the proof in 3 independent parts.

Verification functions produce dependent type.

$\Longrightarrow$ Avoid axioms, improves confidence in preprocessing.

Forced clarification of the semantics:

- Unexpected behaviour (*ex: RETURN after a DEROUTE*),
- Bug (*ex: the FP contains more than 256 blocks/stages*).

$\Longrightarrow$ 2,100 loc of OCaml and 20,000 loc of Coq (14% of functional code).

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The C **mathematical library** verified using **Frama-C**

- Verification of the absence of runtime errors in the library,
- Verification of functional properties on some floating-point functions.

A **flight plan generator** verified using **Coq**

- Development of a new generator in Coq with new features,
- Formalisation of the flight plan semantics,
- Verification of the preservation of the semantics.

Limitation of these verification processes

- Verifications based on hypotheses,
- Require an expertise in formal methods,
- High cost in terms of maintainability.

Future works

Continue the verification of the **mathematical library**:

- Verification of calls to library functions,
- Verifying the floating-point library without the `real` model,

Improve the **new flight plan generator**:

- Verify new properties on the flight plan language,
- Reduce the number of pre-processing steps,
- Generalise the generator.

Verify the **Paparazzi autopilot generator**, similarly to VFPG.

Use **model checking** approaches:

- Verify critical C code using model checking tools such as CBMC,
- Verify design of hardware components.

Formal verification of an UAV autopilot

Static analysis and Verified Code Generation

Case study: Paparazzi

Project publicly available

- **Verified library**: `gitlab.isae-supaero.fr/b.pollien/paparazzi-frama-c`
- **VFPG**: `gitlab.isae-supaero.fr/b.pollien/vfpg`

Publications

- Technical report:
  - Formal verification for autopilots: preliminary state of the start
  - A gentle introduction to C code verification using the Frama-C platform
- Verification of some parts of Paparazzi mathematical library
  Publications: AFADL 2021, FMICS 2021
- Development of a verified flight plan generator
  Publications: FormaliSE 2023

This work is supported by the Defense Innovation Agency (AID) of the French Ministry of Defense
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The **drone environment** can be modelled in a variety of ways.

From the point of view of the flight plan execution, the global drone environment can be split into 2 distinct elements:

- the memory storing the execution **state** of the flight plan,
- the memory that can be modified by flight plan **external functions**.

Remark

External functions can be:

- complex functions that corresponds to navigation stages,
- arbitrary C code contained in the flight plan.

$\Longrightarrow$ **It is not possible to represent the effect of their execution.**

$\Longrightarrow$ **We assume that these 2 memory regions are strictly disjoint.**

The FP semantics will use fp_env, an abstraction of the drone environment.

```
Record fp_env := {
    state: fp_state;
    trace: fp_trace;
}.
```

fp_state represents an abstraction of the current state of the flight plan.

```
Record fp_state := {
    idb:  block_id; stages:  list fp_stages; (* Current position *)
    lidb: block_id; lstages: list fp_stages; (* Last position *)
}
```

A **position** is a couple of a block ID and the remaining stages to execute.

fp_trace represents the history of external functions execution.

```
Variant fp_event := COND (cond * bool) | C_CODE (c: c_code).
Definition fp_trace := list fp_event.
```

Problems faced

Contracts of the trigonometric functions from the libc do not provide mathematical results.

$\Longrightarrow$ Extend the contracts.

ex: Extension of the contract for the function $sinf$.

```
/*@ requires finite_arg: \is_finite(x);
    assigns \result \from x;
    ensures finite_result: \is_finite(\result);
    ensures result_domain: -1. <= \result <= 1.;
    ensures result_value: \result == \sin(x);
*/
extern float sinf(float x);
```

Some lemmas could not be proved by the SMT solvers.

$\Longrightarrow$ Enable interactive mode of Frama-C to use Coq.

Examples of lemmas proved with Coq

- Lemma to verify the correctness of the function `quat_of_rmat`

$$
\begin{aligned}
&\forall R \in SO_3(\mathbb{R}), \forall q \in \mathbb{H},\\
&\qquad ||q|| > 0 \wedge Tr(R) > 0\\
&\qquad \rightarrow (R = \texttt{rmat\_of\_quat}(q) \leftrightarrow q = \texttt{quat\_of\_rmat}(R))
\end{aligned}
$$

- Lemma used to verify that `rmat_of_euler` compute rotation matrix:

$$
\begin{aligned}
&\forall a, b, c \in \mathbb{R},\\
&\qquad \sin(a)^2 \cos(b)^2\\
&\qquad + (\sin(a)\sin(b)\cos(c) - \sin(c)\cos(a))^2\\
&\qquad + (\cos(c)\cos(a) + \sin(a)\sin(b)\sin(c))^2 = 1
\end{aligned}
$$